1 n is the middle integer of three consecutive positive integers.

The three integers are multiplied to give a product.

n is then added to the product.

Prove that the result is a cube number.

[4 marks]

let 3 integers to be : $n-1$, n , $n+1$ (1)

$(n-1)(n+1) = n^2-1$ (1)

$n(n^2-1) = n^3-n$

$n^3-n+n = n^3$ (1)

(1)

2 x is an integer.

Prove that $35 + (3x + 1)^2 - 2x(4x - 3)$ is a square number.

[4 marks]

$(3x+1)^2 = 9x^2 + 6x + 1$

$2x(4x-3) = 8x^2 - 6x$ (1)

$35 + 9x^2 + 6x + 1 - 8x^2 + 6x$

$= 9x^2 - 8x^2 + 6x + 6x + 36$ (1)

$x^2 + 12x + 36$ (1)

$(x+6)(x+6)$

$= (x+6)^2$ (1)

3 A journey has two stages.

	Distance (km)	Average speed (km/h)	Time (h)
Stage 1	30	a	$\frac{30}{a}$
Stage 2	30	b	$\frac{30}{b}$

Show that the average speed for the **whole** journey, in km/h, is $\frac{2ab}{a+b}$

[3 marks]

total time $= \frac{30}{a} + \frac{30}{b} = \frac{30a + 30b}{ab}$ (1)

total distance $= 30 + 30 = 60$

average speed $= \frac{60(ab)}{30a + 30b}$ (1) $= \frac{30(2ab)}{30(a+b)}$

$= \frac{2ab}{a+b}$ (1)

4 A factory packs x boxes of teabags per hour.

Each box contains 80 teabags.

Show that the factory packs $\frac{4x}{3}$ teabags per minute.

[2 marks]

$80 \times x = 80x$ (1)

$\frac{80x}{1 \text{ hour}} \times \frac{1 \text{ hour}}{60 \text{ mins}} = \frac{80}{60} x$ per mins (1)

$= \frac{4x}{3}$ per mins

5 Two integers have a difference of 6

The integers are multiplied together.

9 is then added.

Prove algebraically that the result is always a square number.

[3 marks]

let 2 integers = n and $n+6$

$n(n+6) = n^2 + 6n$ (1)

$= n^2 + 6n(+9)$ (1)

$= (n+3)(n+3)$

$= (n+3)^2$ (1)